Question 1: Let integers x, y satisfy $x^2 - y^2 = 2019$. Which statement is correct?

a) No integer solutions exist.

b) Solutions correspond to factor pairs of 2019 with x = $x = \frac{u+v}{2},\ y = \frac{u-v}{2}$ where uv = 2019 and u,v have same parity.

c) x must be prime.

d) y must be multiple of 2019.

Question 2: A witness is correct 80% of the time. You have only that witness identifying a suspect. Best action for investigator?

a) Treat witness as definitive proof.

b) Use witness as provisional lead but seek independent corroboration (forensic evidence, alibi checks).

c) Dismiss the witness completely.

d) Arrest immediately without further evidence.

Question 3: Which identity is always true for all real a,b?

a) $(a+b)^2 = a^2 + 2ab + b^2$

b) $(a+b)^3 = a^3 + b^3$

c) $a^2 - b^2 = (a - b)^2$

d) $(a - b)^2 = a^2 - b^2$

Question 4: Quarterly sales: Q1=1000. Q2 = Q1 × 1.25. Q3 = Q2 × 0.90. Q4 = Q3 × 1.20. Total yearly sales = ?

a) 4200

b) 4725

c) 4400

d) 4600

Question 5: Which statement about rationals/irrationals is always true?

a) Product of two irrationals is irrational.

b) Sum of a rational and an irrational is irrational.

c) Square of any irrational is irrational.

d) All irrational numbers are algebraic.

Question 6: Twelve coins, one counterfeit heavier or lighter unknown. With 3 balance weighings you can:

a) Always determine which coin and whether heavier or lighter

b) Never identify.

c) Identify coin but not whether heavier or lighter.

d) Identify heavier only.

Question 7: Polynomial p (x) = $x^4 + ax^3 + bx^2 + cx + d$ has roots in arithmetic progression. Which is true?

a) Sum of roots = −a and the roots are symmetric about their average → Vieta relations impose symmetry constraints.

b) b must equal 0 always.

c) a must equal 0.

d) No constraints at all.

Question 8: A 100-L mixture has A:B = 3:2. 10 L of the mixture are removed and replaced by 10 L of pure B. What is the new ratio A:B?

a) 27 : 23

b) 9 : 11

c) 3 : 2

d) 11 : 9

Question 9: Best general approach to solve inequality with absolute values like $|x-2| + |x+1| < 5$?

a) Square both sides.

b) Split into cases by sign of expressions inside absolute values and solve each.

c) Multiply out absolute values.

d) Use only graphing.

Question 10: Two meta-analyses disagree. Editor should:

a) Publish neither.

b) Publish both with methodological comparison and commentary explaining reasons for divergence.

c) Pick the one with favourable conclusion.

d) Average their results numerically without review.

Question 11: Multiplicative order of 2 modulo 31 (small prime): smallest k > 0 with $2^k \equiv 1 \pmod{31}$ is:
a) 5
b) 30
c) 10
d) 15

Question 12: Distribution strongly right skewed; which central measure to report?
a) Mean
b) Median
c) Mode
d) Range

Question 13: For positive real x, which inequality holds: $x + \frac{1}{x} \geq 2$. The equality occurs at:
a) x = 1
b) x = 2
c) $x = \frac{1}{2}$
d) none of the above

Question 14: Statements: S1 "Exactly one of S2, S3 is true." S2 "S3 is false." S3 "S1 is false." Which assignment is consistent?
a) S1 true, S2 false, S3 false
b) S1 false, S2 true, S3 false
c) S1 true, S2 true, S3 false
d) No consistent assignment

Question 15: Which inequality always true for real x,y?
a) $(x-y)^2 \geq 0$
b) $(x-y)^2 \leq 0$
c) $xy \geq x + y$
d) $x^2 + y^2 \leq (x + y)^2$

Question 16: Correlation = 0.95 between two variables in observational data. Best interpretation?
a) Strong linear association but not proof of causation; investigate confounding and data quality.
b) Causation established.
c) Variables identical.
d) Ignore.

Question 17: Evaluate the finite continued fraction 1 / (1+1 / (1+1)) Value =
a) 2/3
b) 3/5
c) 3/4
d) 5/8

Question 18: A sudden single-day spike appears in recorded sensor data. First step:
a) Check instrumentation, calibration logs, and raw entries before inferring phenomenon.
b) Publish spike as discovery immediately.
c) Discard dataset.
d) Smooth data and ignore.

Question 19: If a, b, c > 0 are in GP with b = 6 and a + c = 12, find a and c.
a) 6 and 6
b) 4 and 8
c) 3 and 9
d) 2 and 10

Question 20: Minimize $f(x) = |x - 1| + |x - 4|$. The minimizer set is:
a) Any real x
b) Any x in [1,4] (any median of {1,4})
c) x = 2 only
d) x = 0 only

Question 21: Recurrence $a_{n+1} = 3a_n + 2$ with $a_0 = 1$ Closed form:
a) $2 \cdot 3^{n-1}$
b) $3^{n+1} - 1$
c) $3^n + 1$
d) $3^n - 1$

Question 22: Running many hypothesis tests at $\alpha = 0.05$ without correction risks:
a) Inflated Type I error
b) More power automatically.
c) Type II error inflation only.
d) No risk.

Question 23: Which number is irrational?
a) $\sqrt{2}$
b) 22/7
c) 1/2
d) 9/3

Question 24: With weights 1, 3, 9, 27, 81 on a balance (weights may be placed on either pan), maximum integer weight exactly measurable is:

a) 121

b) 243

c) 40

d) 242

Question 25: Given x + y + z = 3 and $x^2 + y^2 + z^2 = 5$. Then xy + yz + zx equals:

a) 1

b) 2

c) 4

d) 0

Question 26: For bivariate data with outliers, which correlation measure is more robust?

a) Pearson correlation

b) Spearman rank correlation

c) Cosine similarity

d) Mean difference

Question 27: Roots of polynomial $x^3 - 7x + 6$ are:

a) 1,2,-3

b) 1,-2,3

c) -1,-2,10

d) 2,2,3

Question 28: An effect estimate with 95% CI (−2%, 22%). Conclusion?

a) Effect possibly positive but not statistically significant at α = 0.05;

b) Statistically significant.

c) Definitely harmful.

d) Mandatory policy change.

Question 29: Evaluate $\sum_{k=1}^{n} k(k+1)$ =

a) $\frac{n(n+1)(n+2)}{3}$

b) No closed form.

c) Use integration only.

d) Stirling formula.

Question 30: Train A covers 120 km at 60 km/h. Train B covers 150 km at 75 km/h. Which train reaches first?

a) Train A

b) Train B

c) Both reach at the same time

d) Cannot be determined

Question 31: Best method to solve integer linear Diophantine ax + by = c:

a) Extended Euclidean algorithm to find one solution and parameterize all.

b) Quadratic formula.

c) Matrix inversion over reals.

d) No solution method.

Question 32: Three boxes labelled apples, oranges, mixed; all labels wrong. You open one fruit from box labelled “mixed.” Minimal strategy?

a) Take a fruit from box labelled “mixed”, deduce contents, and then relabel all boxes accordingly.

b) Open all boxes.

c) Choose randomly.

d) Cannot deduce.

Question 33: Five friends are sitting in a row. A is to the right of B, but to the left of C. D is to the left of B but to the right of E. Who is sitting in the middle?

a) A

b) B

c) C

d) D

Question 34: In a class of 80 students, 48 are boys. If 50% of the boys and 75% of the girls passed an exam, how many students in total passed?

a) 46

b) 48

c) 50

d) 62

Question 35: Geometric sum:

a) Multiply by ratio then subtract original to eliminate terms.

b) Use only differentiation.

c) Counting principle only.

d) Prime factorization.

Question 36: Compare areas: square and equilateral triangle with same perimeter P. Which has larger area?

a) Square has larger area.
b) Triangle has larger area.
c) Areas equal.
d) Depends on P.

Question 37: If $2^n – 1$ is prime, then:

a) n must be prime
b) n can be composite.
c) n must be even.
d) no relation.

Question 38: Partition problem for n ≈ 30: competent approach?

a) Use meet-in-the-middle or pseudo - polynomial DP (subset-sum) rather than pure brute force.
b) Greedy always optimal.
c) No method exists.
d) Random guessing.

Question 39: Let ω be primitive cube root of unity. Compute $1 + \omega + \omega^2$

a) 0
b) 1
c) 3
d) −1

Question 40: Significant χ^2 in contingency table — prudent next step:

a) Examine cell contributions, check assumptions (expected counts), and estimate effect sizes.
b) Declare causation.
c) Ignore cell counts.
d) Merge categories arbitrarily.

Question 41: Inequality $(\sum a_i)^2 \le n \sum a\frac{2}{i}$ is a form of:

a) Cauchy–Schwarz inequality
b) Bernoulli inequality
c) Triangle inequality
d) Euler's inequality

Question 42: For a logic grid puzzle, best approach is:

a) Translate clues to constraints, use elimination/backtracking search to find consistent assignment.
b) Random guessing.
c) Linear algebra.
d) Graph colouring only.

Question 43: If $x^2 + y^2 + z^2 = 1$ with $x, y, z \ge 0$, the maximum of $x + y + z$ is:

a) √3
b) 1
c) 3
d) 2

Question 44: Data Missing Not At Random (MNAR): most cautious approach:

a) Model missingness mechanism explicitly, perform sensitivity analyses, and consider multiple imputation under plausible assumptions.
b) Mean imputation.
c) Delete dataset.
d) Fill zeros.

Question 45: Series $\sum (-1)^n/n$ converges by which test?

a) Alternating series (Leibniz) test
b) Ratio test.
c) Divergence test.
d) Root test.

Question 46: Triangle sides 13,14,15: area equals:

a) 84
b) 90
c) 78
d) 72

Question 47: Fast divisibility test for 11:

a) Alternating sum of digits
b) Sum of digits.
c) Multiply digits.
d) Test mod 9.

Question 48: Take 1–3 stones per move, last wins. With 21 stones, first-player winning move?

a) Take 1 to leave 20 (a multiple of 4) and then mirror opponent to win.
b) Take 2 only.
c) Take 3 only.
d) First player loses with optimal play.

Question 49: For nonnegative numbers, arithmetic mean A and geometric mean G satisfy:

a) $A \geq G$, with equality if all numbers equal.
b) $A \leq G$.
c) No general relation.
d) $A = G$ always.

Question 50: Ignoring autocorrelation in regression residuals commonly leads to:

a) OLS coefficient estimates unbiased but standard errors incorrect → inference invalid; use GLS or robust SE.
b) OLS unbiasedness lost always.
c) OLS becomes nonlinear.
d) No consequence.

Question 51: Statements:

1. **All books are papers.**
2. **Some papers are pens.**

Conclusions:
I. Some pens are books.
II. Some papers are books.

a) Only I follows
b) Only II follows
c) Both I and II follow
d) Neither I nor II follows

Question 52:100 prisoners toggle bulb: number of bulbs on equals:

a) Number of perfect squares ≤ 100
b) Number of primes ≤ 100.
c) 50.
d) 25.

Question 53: Quadratic with discriminant $\Delta < 0$ has:

a) Two complex conjugate roots, no real roots.
b) Two distinct real roots.
c) One repeated real root.
d) Only irrational roots.

Question 54: Farmer fences rectangle adjacent to river (no fence along river). For fixed total fencing L, rectangle maximizing area has dimensions x (perp to river) and y (along river) with: $2x + y = L$. Optimal x and y are:

a) $x = L/4$, $y = L/2$.
b) square $x = y$.
c) $x \to 0$ narrow strip.
d) $y \to 0$.

Question 55: Why is testing primality for large n computationally significant?

a) Because factoring large composites is hard; fast deterministic primality and difficulty of factoring underpin cryptography.
b) Primality trivial by summing digits.
c) All large numbers are prime.
d) No relation to cryptography.

Question 56: Compare volatility across years: best test?

a) F-test or Levene's test for equality of variances (Levene if non-normal) plus visual checks.
b) t-test for means.
c) Chi-square test for independence.
d) Correlation test.

Question 57: Solving integer linear system $Ax = b$ (Diophantine) typically uses:

a) Smith normal form or lattice basis reduction to find integer solutions.
b) Only Gaussian elimination over reals.
c) No algorithm exists.
d) Random guessing.

Question 58: Five consecutive integers sum to zero. Which are they?

a) −2,−1,0,1,2
b) −1,0,1,2,3
c) 0,1,2,3,4
d) −3,−2,−1,0,1

Question 59: For independent events A and B, $P(A \cup B)$ =

a) $P(A) + P(B) - P(A)\,P(B)$

b) $P(A) + P(B)$

c) $P(A)\,P(B)$

d) $\max(P(A), P(B))$

Question 60: Continuous f on [0,1] that changes sign (positive somewhere and negative somewhere) guarantees by Intermediate Value Theorem:

a) There exists c in (0,1) with f(c) = 0.

b) f positive everywhere.

c) f negative everywhere.

d) No guaranteed root.

ROUGH WORK
(NOT TO BE EVALUATED)

DATE

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