

Question 1: Recurrence $a_{n+1} = 3a_n + 2$ with $a_0 =$

1 Closed form:

- a) $2 \cdot 3^{n-1}$
- b) $3^{n+1} - 1$
- c) $3^n + 1$
- d) $3^n - 1$

Question 2: Running many hypothesis tests at $\alpha = 0.05$ without correction risks:

- a) Inflated Type I error
- b) More power automatically.
- c) Type II error inflation only.
- d) No risk.

Question 3: Which number is irrational?

- a) $\sqrt{2}$
- b) $22/7$
- c) $1/2$
- d) $9/3$

Question 4: With weights 1, 3, 9, 27, 81 on a balance (weights may be placed on either pan), maximum integer weight exactly measurable is:

- a) 121
- b) 243
- c) 40
- d) 242

Question 5: Given $x + y + z = 3$ and $x^2 + y^2 + z^2 = 5$. Then $xy + yz + zx$ equals:

- a) 1
- b) 2
- c) 4
- d) 0

Question 6: In a class of 80 students, 48 are boys. If 50% of the boys and 75% of the girls passed an exam, how many students in total passed?

- a) 46
- b) 48
- c) 50
- d) 62

Question 7: Geometric sum:

- a) Multiply by ratio then subtract original to eliminate terms.
- b) Use only differentiation.
- c) Counting principle only.
- d) Prime factorization.

Question 8: Compare areas: square and equilateral triangle with same perimeter P. Which has larger area?

- a) Square has larger area.
- b) Triangle has larger area.
- c) Areas equal.
- d) Depends on P.

Question 9: If $2^n - 1$ is prime, then:

- a) n must be prime
- b) n can be composite.
- c) n must be even.
- d) no relation.

Question 10: Partition problem for $n \approx 30$: competent approach?

- a) Use meet-in-the-middle or pseudo - polynomial DP (subset-sum) rather than pure brute force.
- b) Greedy always optimal.
- c) No method exists.
- d) Random guessing.

Question 11: Let ω be primitive cube root of unity. Compute $1 + \omega + \omega^2$

- a) 0
- b) 1
- c) 3
- d) -1

Question 12: Ignoring autocorrelation in regression residuals commonly leads to:

- a) OLS coefficient estimates unbiased but standard errors incorrect $\rightarrow$ inference invalid; use GLS or robust SE.
- b) OLS unbiasedness lost always.
- c) OLS becomes nonlinear.
- d) No consequence.

Question 13: Statements:

1. All books are papers.
2. Some papers are pens.

Conclusions:

- I. Some pens are books.
- II. Some papers are books.

- a) Only I follows
- b) Only II follows
- c) Both I and II follow
- d) Neither I nor II follows

Question 14: A 100 prisoners toggle bulb: number of bulbs on equals:

- a) Number of perfect squares ≤ 100
- b) Number of primes ≤ 100 .
- c) 50.
- d) 25.

Question 15: Quadratic with discriminant $\Delta < 0$ has:

- a) Two complex conjugate roots, no real roots.
- b) Two distinct real roots.
- c) One repeated real root.
- d) Only irrational roots.

Question 16: Farmer fences rectangle adjacent to river (no fence along river). For fixed total fencing L, rectangle maximizing area has dimensions x (perp to river) and y (along river) with: $2x + y = L$. Optimal x and y are:

- a) $x = L/4$, $y = L/2$.
- b) square $x = y$.
- c) $x \rightarrow 0$ narrow strip.
- d) $y \rightarrow 0$.

Question 17: Why is testing primality for large n computationally significant?

- a) Because factoring large composites is hard; fast deterministic primality and difficulty of factoring underpin cryptography.
- b) Primality trivial by summing digits.
- c) All large numbers are prime.
- d) No relation to cryptography.

Question 18: Compare volatility across years: best test?

- a) F-test or Levene's test for equality of variances (Levene if non-normal) plus visual checks.
- b) t-test for means.
- c) Chi-square test for independence.
- d) Correlation test.

Question 19: Solving integer linear system $Ax = b$ (Diophantine) typically uses:

- a) Smith normal form or lattice basis reduction to find integer solutions.
- b) Only Gaussian elimination over reals.
- c) No algorithm exists.
- d) Random guessing.

Question 20: Five consecutive integers sum to zero. Which are they?

- a) $-2, -1, 0, 1, 2$
- b) $-1, 0, 1, 2, 3$
- c) $0, 1, 2, 3, 4$
- d) $-3, -2, -1, 0, 1$

Question 21: For independent events A and B, $P(A \cup B) =$

- a) $P(A) + P(B) - P(A)P(B)$
- b) $P(A) + P(B)$
- c) $P(A)P(B)$
- d) $\max(P(A), P(B))$

Question 22: Continuous f on $[0,1]$ that changes sign (positive somewhere and negative somewhere) guarantees by Intermediate Value Theorem:

- a) There exists c in $(0,1)$ with $f(c) = 0$.
- b) f positive everywhere.
- c) f negative everywhere.
- d) No guaranteed root.

Question 23: Significant χ^2 in contingency table — prudent next step:

- a) Examine cell contributions, check assumptions (expected counts), and estimate effect sizes.
- b) Declare causation.
- c) Ignore cell counts.
- d) Merge categories arbitrarily.

Question 24: Inequality $(\sum a_i)^2 \leq n \sum a_i^2$ is a

form of:

- a) Cauchy–Schwarz inequality
- b) Bernoulli inequality
- c) Triangle inequality
- d) Euler’s inequality

Question 25: For a logic grid puzzle, best approach is:

- a) Translate clues to constraints, use elimination/backtracking search to find consistent assignment.
- b) Random guessing.
- c) Linear algebra.
- d) Graph colouring only.

Question 26: If $x^2 + y^2 + z^2 = 1$ with $x, y, z \geq 0$, the maximum of $x + y + z$ is:

- a) $\sqrt{3}$
- b) 1
- c) 3
- d) 2

Question 27: Data Missing Not At Random (MNAR): most cautious approach:

- a) Model missingness mechanism explicitly, perform sensitivity analyses, and consider multiple imputation under plausible assumptions.
- b) Mean imputation.
- c) Delete dataset.
- d) Fill zeros.

Question 28: Series $\sum (-1)^n/n$ converges by which test?

- a) Alternating series (Leibniz) test
- b) Ratio test.
- c) Divergence test.
- d) Root test.

Question 29: Triangle sides 13,14,15: area equals:

- a) 84
- b) 90
- c) 78
- d) 72

Question 30: Fast divisibility test for 11:

- a) Alternating sum of digits
- b) Sum of digits.
- c) Multiply digits.
- d) Test mod 9.

Question 31: Take 1–3 stones per move, last wins. With 21 stones, first-player winning move?

- a) Take 1 to leave 20 (a multiple of 4) and then mirror opponent to win.
- b) Take 2 only.
- c) Take 3 only.
- d) First player loses with optimal play.

Question 32: For nonnegative numbers, arithmetic mean A and geometric mean G satisfy:

- a) $A \geq G$, with equality if all numbers equal.
- b) $A \leq G$.
- c) No general relation.
- d) $A = G$ always.

Question 33: For bivariate data with outliers, which correlation measure is more robust?

- a) Pearson correlation
- b) Spearman rank correlation
- c) Cosine similarity
- d) Mean difference

Question 34: Roots of polynomial $x^3 - 7x + 6$ are:

- a) 1,2,-3
- b) 1,-2,3
- c) -1,-2,10
- d) 2,2,3

Question 35: An effect estimate with 95% CI (-2%, 22%). Conclusion?

- a) Effect possibly positive but not statistically significant at $\alpha = 0.05$;
- b) Statistically significant.
- c) Definitely harmful.
- d) Mandatory policy change.

Question 36: Evaluate $\sum_{k=1}^n k(k+1) =$

a) $\frac{n(n+1)(n+2)}{3}$

- b) No closed form.
- c) Use integration only.
- d) Stirling formula.

Question 37: Train A covers 120 km at 60 km/h. Train B covers 150 km at 75 km/h. Which train reaches first?

- a) Train A
- b) Train B
- c) Both reach at the same time
- d) Cannot be determined

Question 38: Best method to solve integer linear Diophantine $ax + by = c$:

- a) Extended Euclidean algorithm to find one solution and parameterize all.
- b) Quadratic formula.
- c) Matrix inversion over reals.
- d) No solution method.

Question 39: Three boxes labelled apples, oranges, mixed; all labels wrong. You open one fruit from box labelled "mixed." Minimal strategy?

- a) Take a fruit from box labelled "mixed", deduce contents, and then relabel all boxes accordingly.
- b) Open all boxes.
- c) Choose randomly.
- d) Cannot deduce.

Question 40: Five friends are sitting in a row. A is to the right of B, but to the left of C. D is to the left of B but to the right of E. Who is sitting in the middle?

- a) A
- b) B
- c) C
- d) D

Question 41: Multiplicative order of 2 modulo 31 (small prime): smallest $k > 0$ with $2^k \equiv 1 \pmod{31}$ is:

- a) 5
- b) 30
- c) 10
- d) 15

Question 42: Distribution strongly right skewed; which central measure to report?

- a) Mean
- b) Median
- c) Mode
- d) Range

Question 43: For positive real x , which inequality holds: $x + \frac{1}{x} \geq 2$. The equality occurs at:

- a) $x = 1$
- b) $x = 2$
- c) $x = \frac{1}{2}$
- d) none of the above

Question 44: Statements: S1 "Exactly one of S2, S3 is true." S2 "S3 is false." S3 "S1 is false." Which assignment is consistent?

- a) S1 true, S2 false, S3 false
- b) S1 false, S2 true, S3 false
- c) S1 true, S2 true, S3 false
- d) No consistent assignment

Question 45: Which inequality always true for real x, y ?

- a) $(x-y)^2 \geq 0$
- b) $(x-y)^2 \leq 0$
- c) $xy \geq x + y$
- d) $x^2 + y^2 \leq (x + y)^2$

Question 46: Correlation = 0.95 between two variables in observational data. Best interpretation?

- a) Strong linear association but not proof of causation; investigate confounding and data quality.
- b) Causation established.
- c) Variables identical.
- d) Ignore.

Question 47: Evaluate the finite continued fraction $1 / (1 + 1 / (1 + 1))$ Value =

- a) $2/3$
- b) $3/5$
- c) $3/4$
- d) $5/8$

Question 48: A sudden single-day spike appears in recorded sensor data. First step:

- a) Check instrumentation, calibration logs, and raw entries before inferring phenomenon.
- b) Publish spike as discovery immediately.
- c) Discard dataset.
- d) Smooth data and ignore.

Question 49: If $a, b, c > 0$ are in GP with $b = 6$ and $a + c = 12$, find a and c .

- a) 6 and 6
- b) 4 and 8
- c) 3 and 9
- d) 2 and 10

Question 50: Minimize $f(x) = |x - 1| + |x - 4|$. The minimizer set is:

- a) Any real x
- b) Any x in $[1, 4]$ (any median of $\{1, 4\}$)
- c) $x = 2$ only
- d) $x = 0$ only

Question 51: Let integers x, y satisfy $x^2 - y^2 = 2019$. Which statement is correct?

- a) No integer solutions exist.
- b) Solutions correspond to factor pairs of 2019 with $x = \frac{u+v}{2}$, $y = \frac{u-v}{2}$ where $uv = 2019$ and u, v have same parity.
- c) x must be prime.
- d) y must be multiple of 2019.

Question 52: A witness is correct 80% of the time. You have only that witness identifying a suspect. Best action for investigator?

- a) Treat witness as definitive proof.
- b) Use witness as provisional lead but seek independent corroboration (forensic evidence, alibi checks).
- c) Dismiss the witness completely.
- d) Arrest immediately without further evidence.

Question 53: Which identity is always true for all real a, b ?

- a) $(a+b)^2 = a^2 + 2ab + b^2$
- b) $(a+b)^3 = a^3 + b^3$
- c) $a^2 - b^2 = (a - b)^2$
- d) $(a - b)^2 = a^2 - b^2$

Question 54: Quarterly sales: $Q_1 = 1000$. $Q_2 = Q_1 \times 1.25$. $Q_3 = Q_2 \times 0.90$. $Q_4 = Q_3 \times 1.20$.

Total yearly sales = ?

- a) 4200
- b) 4725
- c) 4400
- d) 4600

Question 55: Which statement about rationals/irrationals is always true?

- a) Product of two irrationals is irrational.
- b) Sum of a rational and an irrational is irrational.
- c) Square of any irrational is irrational.
- d) All irrational numbers are algebraic.

Question 56: Twelve coins, one counterfeit heavier or lighter unknown. With 3 balance weighings you can:

- a) Always determine which coin and whether heavier or lighter
- b) Never identify.
- c) Identify coin but not whether heavier or lighter.
- d) Identify heavier only.

Question 57: Polynomial $p(x) = x^4 + ax^3 + bx^2 + cx + d$ has roots in arithmetic progression. Which is true?

- a) Sum of roots $= -a$ and the roots are symmetric about their average $\rightarrow$ Vieta relations impose symmetry constraints.
- b) b must equal 0 always.
- c) a must equal 0.
- d) No constraints at all.

Question 58: A 100-L mixture has $A:B = 3:2$. 10 L of the mixture are removed and replaced by 10 L of pure B. What is the new ratio $A:B$?

- a) 27 : 23
- b) 9 : 11
- c) 3 : 2
- d) 11 : 9

Question 59: Best general approach to solve inequality with absolute values like $|x-2| + |x+1| < 5$?

- a) Square both sides.
- b) Split into cases by sign of expressions inside absolute values and solve each.
- c) Multiply out absolute values.
- d) Use only graphing.

Question 60: Two meta-analyses disagree.

Editor should:

- a) Publish neither.
- b) Publish both with methodological comparison and commentary explaining reasons for divergence.
- c) Pick the one with favourable conclusion.
- d) Average their results numerically without review.

ROUGH WORK

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